Contents

Preface **vii**

1 Limitations of classic statistics and motivation **1**
1.1 Limitations of classic statistics 1
1.1.1 Mean 1
1.1.2 Unbiasedness 4
1.1.3 Limitations of equal-tail statistical inference 5
1.2 The rationale for a new statistical theory 5
1.3 Motivating example: normal variance 7
1.3.1 Confidence interval for the normal variance 7
1.3.2 Hypothesis testing for the variance 12
1.3.3 MC and MO estimators of the variance 13
1.3.4 Sample size determination for variance 14
1.4 Neyman-Pearson lemma and its extensions 16
1.4.1 Introduction 16
1.4.2 Two lemmas 18

2 Maximum concentration statistics **29**
2.1 Assumptions 29
2.2 Short confidence interval and MC estimator 32
2.3 Density level test 39
2.4 Efficiency and the sufficient statistic 42
2.5 Parameter is positive or belongs to a finite interval 45
2.5.1 Parameter is positive 45
2.5.2 Parameter belongs to a finite interval 48

3 Mode statistics **51**
3.1 Unbiased test 51
3.2 Unbiased CI and MO estimator 56
3.3 Cumulative information and the sufficient statistic 59

4 P-value and duality **65**
4.1 P-value for the double-sided hypothesis . . . 65
4.1.1 General definition . . . 66
4.1.2 P-value for normal variance . . . 70
4.2 The overall powerful test . . . 75
4.3 Duality: converting the CI into a hypothesis test . . . 81
4.4 Bypassing assumptions . . . 82
4.5 Overview . . . 84

5 M-statistics for major statistical parameters **89**
5.1 Exact statistical inference for standard deviation . . . 89
5.1.1 MC-statistics . . . 90
5.1.2 MC-statistics on the log scale . . . 92
5.1.3 MO-statistics . . . 92
5.1.4 Computation of the p-value . . . 93
5.2 Pareto distribution . . . 93
5.2.1 Confidence intervals . . . 94
5.2.2 Hypothesis testing . . . 97
5.3 Coefficient of variation for lognormal distribution . . . 99
5.4 Statistical testing for two variances . . . 101
5.4.1 Computation of the p-value . . . 103
5.4.2 Optimal sample size . . . 103
5.5 Inference for two-sample exponential distribution . . . 105
5.5.1 Unbiased statistical test . . . 106
5.5.2 Confidence intervals . . . 109
5.5.3 The MC estimator of ν . . . 110
5.6 Effect size and coefficient of variation . . . 111
5.6.1 Effect size . . . 112
5.6.2 Coefficient of variation . . . 118
5.6.3 Double-sided hypothesis tests . . . 124
5.6.4 Multivariate ES . . . 124
5.7 Binomial probability . . . 125
5.7.1 The MCL estimator . . . 126
5.7.2 The MCL2 estimator . . . 128
5.7.3 The MCL2 estimator of p^n . . . 129
5.7.4 Confidence interval on the double-log scale . . . 130
5.7.5 Equal-tail and unbiased tests . . . 132
5.8 Poisson rate . . . 135
5.8.1 Two-sided short CI on the log scale . . . 136
5.8.2 Two-sided tests and p-value . . . 138
5.8.3 The MCL estimator of the rate parameter . . . 140
5.9 Meta-analysis model . . . 140
5.9.1 CI and MCL estimator . . . 145

5.10 M-statistics for the correlation coefficient . . . 147
5.10.1 MC and MO estimators . . . 148
5.10.2 Equal-tail and unbiased tests . . . 151
5.10.3 Power function and p-value . . . 153
5.10.4 Confidence intervals . . . 154
5.11 The square multiple correlation coefficient . . . 157
5.11.1 Unbiased statistical test . . . 158
5.11.2 Computation of p-value . . . 161
5.11.3 Confidence intervals . . . 161
5.11.4 The two-sided CI on the log scale . . . 163
5.11.5 The MCL estimator . . . 164
5.12 Coefficient of determination for linear model . . . 166
5.12.1 CoD and multiple correlation coefficient . . . 167
5.12.2 Unbiased test . . . 168
5.12.3 The MCL estimator for CoD . . . 169

6 Multidimensional parameter 177
6.1 Density level test . . . 177
6.2 Unbiased test . . . 180
6.3 Confidence region dual to the DL test . . . 181
6.4 Unbiased confidence region . . . 184
6.5 Simultaneous inference for normal mean and standard deviation . . 186
6.5.1 Statistical test . . . 187
6.5.2 Confidence region . . . 192
6.6 Exact confidence inference for parameters of the beta distribution . 194
6.6.1 Statistical tests . . . 195
6.6.2 Confidence regions . . . 197
6.7 Two-sample binomial probability . . . 199
6.7.1 Hypothesis testing . . . 199
6.7.2 Confidence region . . . 202
6.8 Exact and profile statistical inference for nonlinear regression . . . 204
6.8.1 Statistical inference for the whole parameter . . . 205
6.8.2 Statistical inference for an individual parameter of interest via profiling . . . 210

Index 221